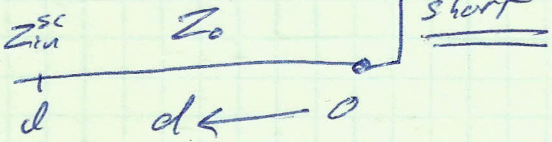
Special Cases of Lossless Line

From Last lectures:

$$\tilde{V}(z) = V_0^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)$$



at $z = -d$ (remember, book uses d as substitute variable, not line length)

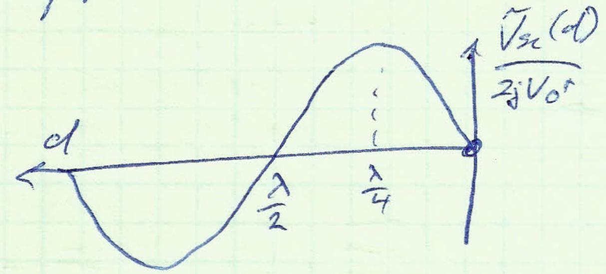
- assume short circuit $Z_L = 0$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

$$\tilde{V}_{sc}(d) = V_0^+\left[e^{j\beta d} - e^{-j\beta d}\right] = 2jV_0^+ \sin\beta d$$

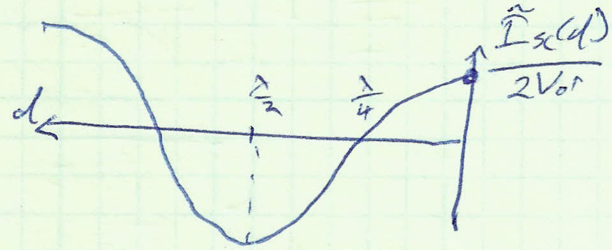
$$\tilde{I}_{sc}(d) = \frac{V_0^+}{Z_0}\left[e^{j\beta d} + e^{-j\beta d}\right] = \frac{2V_0^+}{Z_0}\cos\beta d$$

$$Z_{sc}(d) = \frac{\tilde{V}_{sc}(d)}{\tilde{I}_{sc}(d)} = jZ_0 \tan\beta d$$



for line of length l

$$\boxed{Z_{in}^{sc} = jZ_0 \tan\beta l}$$



- Equivalent Inductance

$$L_{eq} = \frac{Z_0 \tan\beta l}{\omega} \leftarrow \text{use}$$

$z = R + jX = Z_{in}^{sc}$

$j\omega L_{eq} = jZ_0 \tan\beta l$

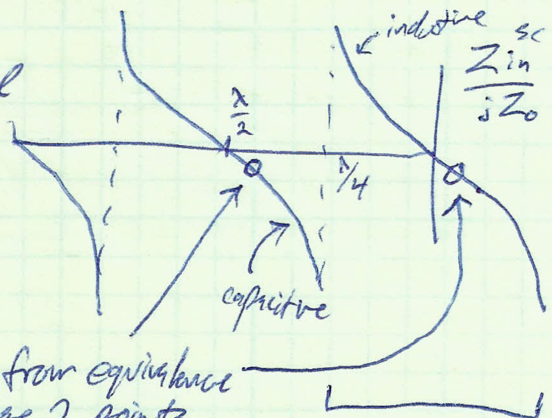
$$l = \frac{1}{\beta}\tan^{-1}\left(\frac{\omega L_{eq}}{Z_0}\right)$$

- Equivalent Capacitance
(for lines longer than $\lambda/4$)

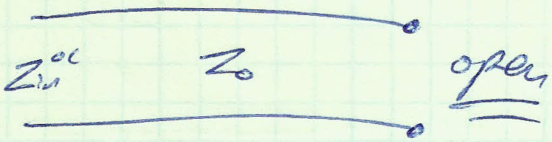
$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan\beta l$$

$$C_{eq} = \frac{-1}{Z_0 \omega \tan\beta l}$$

$$l = \frac{1}{\beta}\left[\pi - \tan^{-1}\left(\frac{1}{\omega C_{eq} Z_0}\right)\right]$$



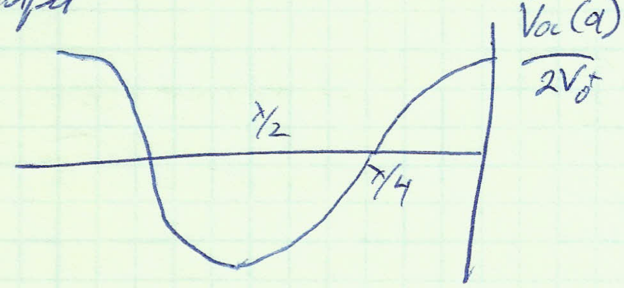
For open-circuited line

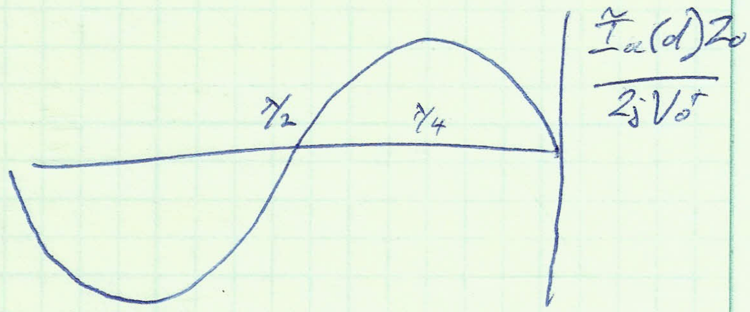


$$\tilde{V}_{oc}(d) = V_0^+\left[e^{j\beta d} + e^{-j\beta d}\right] = 2V_0^+ \cos\beta d$$

$$\tilde{I}_{oc}(d) = \frac{V_0^+}{Z_0}\left[e^{j\beta d} - e^{-j\beta d}\right] = \frac{2jV_0^+}{Z_0}\sin\beta d$$

$$\boxed{Z_{in}^{oc} = \frac{\tilde{V}_{oc}(l)}{\tilde{I}_{oc}(l)} = -jZ_0\cot\beta l}$$



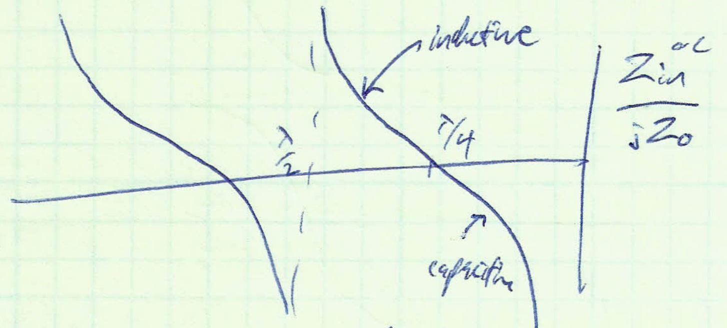


- Note about short/open circuits:

$$Z_0 = \sqrt{Z_{in}^{sc} Z_{in}^{oc}}$$

$$\tan\beta l = \sqrt{\frac{-Z_{in}^{sc}}{Z_{in}^{oc}}}$$

this method can be used to measure Z_0 and β for an unknown transmission line



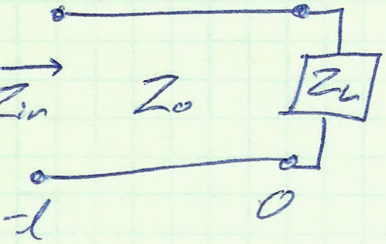
- ~~half-wavelength line~~

~~$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right) = \tan\pi = 0$~~

note the top graphs do not divide to get the bottom one because of the factor of j

- Half-wavelength (or multiples) transmission line

Remember,



$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

when $l = \frac{n\lambda}{2}$ (n = integer)

$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\cdot\frac{n\lambda}{2}\right) = \tan n\pi = 0$$

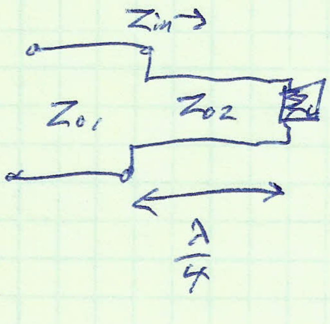
$Z_{in} = Z_L \rightarrow$ transmission line has no effect on impedance

- Quarter-wavelength transformer

when $l = \frac{\lambda}{4}$ (or $\frac{\lambda}{4} + \frac{n\lambda}{2}$)

$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\right) = \tan\frac{\pi}{2} = \infty$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$



To cancel reflections,

$$\frac{Z_{02}^2}{Z_L} = Z_{01}$$

$$Z_{02} = \sqrt{Z_L Z_{01}^2}$$

Power Flow on Lossless Transmission Line

$$\tilde{V}(d) = V_0^+\left(e^{j\beta d} + \Gamma e^{-j\beta d}\right)$$

$$\tilde{I}(d) = \frac{V_0^+}{Z_0}\left(e^{j\beta d} - \Gamma e^{-j\beta d}\right)$$

$$v(d,t) = \mathrm{Re}\left[\tilde{V} e^{j\omega t}\right] = \mathrm{Re}\left[|V_0^+| e^{j\phi^+}\left(e^{j\beta d} + |\Gamma| e^{j\theta_r} e^{-j\beta d}\right) e^{j\omega t}\right]$$

$$= |V_0^+|\left[\cos(\omega t + \beta d + \phi^+) + |\Gamma|\cos(\omega t - \beta d + \phi^+ + \theta_r)\right]$$

$$i(d,t) = \frac{|V_0^+|}{Z_0}\left[\cos(\omega t + \beta d + \phi^+) - |\Gamma|\cos(\omega t - \beta d + \phi^+ + \theta_r)\right]$$

$$P(d,t) = v(d,t)\, i(d,t)$$

$$= \frac{|V_0^+|}{Z_0}\left[\cos^2(\omega t + \beta d + \phi^+) - |\Gamma|^2 \cos^2(\omega t - \beta d + \phi^+ + \theta_r)\right]$$

instantaneous incident power — instantaneous reflected power

use $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$$P^i(d,t) = \frac{|V_0^+|^2}{2Z_0}\left[1 + \cos(2\omega t + 2\beta d + 2\phi^+)\right]$$

and similar expression for $P^r(d,t)$

note: power oscillates at twice the ~~period~~ frequency of the wave.

- Time average power

$$P_{av}^i(d) = \frac{1}{T}\int_0^T P^i(d,t)\,dt = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} P^i(d,t)\,dt$$

Integral over cosine term is 0

We are left with

$$\boxed{P_{av}^i = \frac{|V_0^+|^2}{2Z_0} \qquad P_{av}^r = -|\Gamma|^2 P_{av}^i}$$

Power flowing toward load (and absorbed)

$$P_{av} = P_{av}^i + P_{av}^r = \frac{|V_0^+|^2}{2Z_0}\left[1 - |\Gamma|^2\right]$$

note convention that both power terms are for power flowing in <u>forward</u> direction